6

Pure Probability

The puzzles in this chapter generally require you to make probability calculations. Many of the actual calculations are relatively straightforward, but often it's hard to figure out what calculations to perform because your intuition leads you in the wrong direction. Other puzzles use wording to imply the wrong approach.

Good techniques for solving these kinds of puzzles include figuring out what is actually important, working on the problem incrementally or in steps, and enumerating all of the possible outcomes to see what's actually happening.

(... Pages omitted ...)

PROBABILISTIC CARS

Puzzle: If the probability of a car passing a certain intersection in a 20–minute window is 0.9, what is the probability of a car passing the intersection in a five–minute window? (Assuming the probability is uniform throughout the puzzle.)

The tricky part of this puzzle is figuring out what equation to use. Unless you've seen this sort of problem before, you may not know where to start.

Often when you need to calculate one probability in terms of another, you need to find a way to recast one of the events in terms of the other. In this puzzle, a 20–minute window consists of four five–minute windows.

Exactly how you compose one event from four other events depends on the relationships among the events. In this case, the probability of a car passing within a 20–minute window is the same as the probability of one or more cars passing in any of the four five–minute sub–windows.

Let P be the probability of at least one car passing inside a given five–minute window.

Intuitively you might like to add the probabilities, but adding probabilities isn't that simple because the five–minute sub–windows aren't mutually exclusive. For example, cars could pass in both the first and third five–minute windows. In order to add the probabilities, we would need to subtract out the probabilities of all of the combinations of multiple cars passing during any of the sub–windows, and that sounds like a lot of work.

A better approach is to examine the probabilities of cars *not* passing during the sub–windows. No cars passing during 20 minutes is the same as no cars passing during *any* of the five–minute sub–windows. Those events are independent, so we can multiply their probabilities.

We let P be the probability of a car passing during a five–minute sub–window, so the probability of no car passing is 1–P.

In that case the probability of no car passing during all four of the five–minute sub–windows is $(1–P)^4$.

This is the same as the probability of *no* cars passing during the larger 20–minute window, so the probability of at least one car passing during that window is 1 minus this probability or $1–(1–P)^4$.

The problem statement says that this probability is 0.9, so we get the following equation.

$$0.9=1-(1-P)^4$$

Solving this for P gives:

$$P = 1 - \sqrt[4]{0.1} \approx 0.4377$$

In other words, there's almost a 44% chance that a car will pass in any given five–minute window.

(... More pages omitted ...)